On The Symmetry Of Primes (WIP)

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Abstract

By definition prime number is a natural number >1 that cannot be formed by multiplication of two smaller natural numbers. Thanks to various amazing results produced by humanity so far we know a lot about primes (for instance, thanks to Prime Number Theorem we know that primes are asymptotically distributed among the positive integers), however prime set still hides secrets and confirmation of primality is still a challenge - it is either based on brute force (trial division), or based on probability test (i.e. Miller-Rabin primality test). Even recent breaktrough, AKS, is still too slow to be practical. Faster methods are available for numbers of special forms (i.e. Mersenne primes) but not for all. This work is devoted to search for symmetry in prime set in order to find its interesting properties and locate promising candidates for primes.

1 Problem statement

TBD

2 Sequences under investigation

First group of sequences under investigation is one which is showing balance between primes situated in regular intervals.

Let $p(i)$ be a function which is returning the ith prime. Then let's define m_L $(L \geq 3)$ as follows:

$$m_L = p(k) + p(k+L) = p(k+1) + p(k+L-1) \quad (1)$$

for some k>0.

Lemma 1. *m_L is always positive even number.*

Proof. There is only one even prime: 2 ($p(1)$). All other primes, $p(i), i$>1, are odd. By definition $L \geq 3$ and k>0, thus: $p(k+1) \geq p(2)$, $p(k+L-1) \geq p(3)$ - both parts of this equation are odd primes. This means that their sum, $m_L = p(k+1) + p(k+L-1)$, is even. □

3 Results

Executed results showed:

Table 1: Options - formulas for prime candidate

Option	Formula for $f(m, \Delta)$
1	$f(m, \Delta) = 9.81764596 \times m^{1.09031708} + \Delta$
2	$f(m, \Delta) = 10 \times m + \Delta$
3	$f(m, \Delta) = 9.1 \times m^{1.1} + 10 + \Delta$

Table 2: m_L

L	First terms of sequence
3	18, 24, 30, 36, 60, 84, ...
5	24, 30, 60, 84, 102, 210, 234, ...

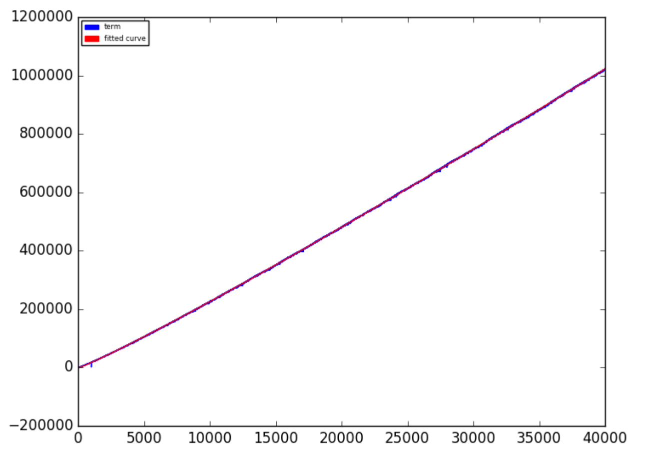


Figure 1: Terms of A333779 and curve of form $a \times x^b + c$ fitted to them ($0 < m < 4 \times 10^4$, $m \in \mathbb{N}$)

Table 3: $a \times x^b + c$ in A333779

Terms	Values of a, b and c
1000	a=7.41655618, b=1.12453747, c=-58.31764747
5000	a=8.59135892 , b=1.10523239, c=-260.5197196
10000	a=9.03416325, b=1.09946314, c=-434.81544801
20000	a=9.67494142, b=1.09211763, c=-884.89244159
30000	a=9.97948888e+00, b=1.08898433e+00 c=-1.23465676e+03
40000	a=1.02739033e+01, b=1.08614446e+00 c=-1.69756790e+03

Table 4: $a \times x^b$ in A333779

$Terms$	a	b
1000	6.96830767	1.13320497
2000	7.39676569	1.12409861
3000	7.71116401	1.1183597
4000	7.88797205	1.11536219
5000	8.12277722	1.111606
6000	8.21458396	1.11020652
7000	8.37057093	1.10790242
8000	8.47069868	1.10646714
9000	8.56506153	1.10515934
10000	8.62159545	1.10438602
11000	8.76414298	1.10248636
12000	8.77573025	1.10233677
13000	8.90075275	1.10072954
14000	8.85437654	1.10131833
15000	9.02766378	1.09915524
16000	9.05616331	1.09880405
17000	9.08023322	1.0985127
18000	9.16007285	1.0975547
19000	9.14854003	1.09769178
20000	9.22652845	1.09677443
21000	9.22652053	1.09677452
22000	9.26771994	1.09629519
23000	9.29016354	1.09603676
24000	9.40337378	1.0947497
25000	9.39419517	1.09485288
26000	9.44486097	1.09428781
27000	9.52108153	1.0934464
28000	9.51475708	1.09351567
29000	9.493371	1.09374966
30000	9.54898231	1.09314533
31000	9.62915882	1.09228312
32000	9.62918219	1.09228284
33000	9.61484865	1.09243537
34000	9.65065324	1.09205596
35000	9.70772857	1.09145545
36000	9.76505854	1.09085744
37000	9.77202974	1.09078514
38000	9.77154319	1.09079016
39000	9.78572873	1.09064415
40000	9.81764596	1.09031708

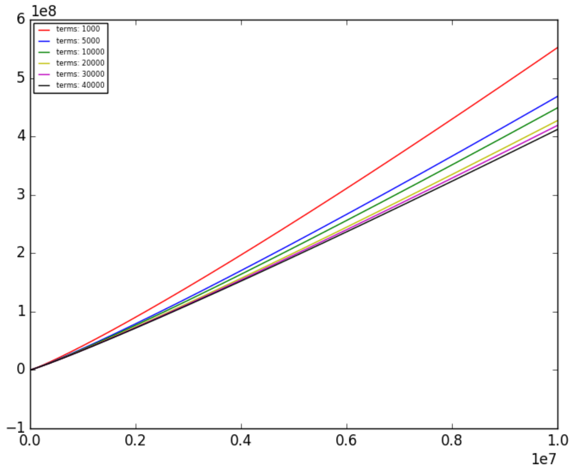
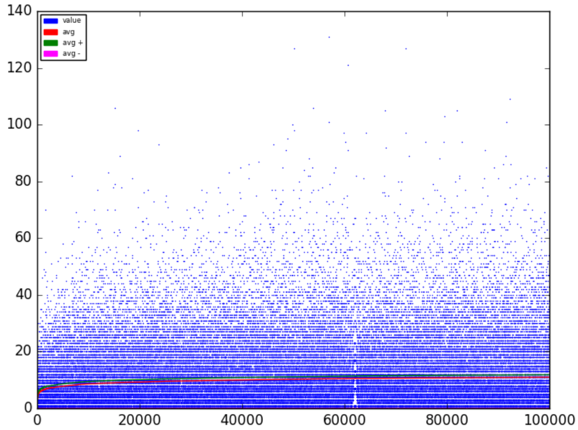
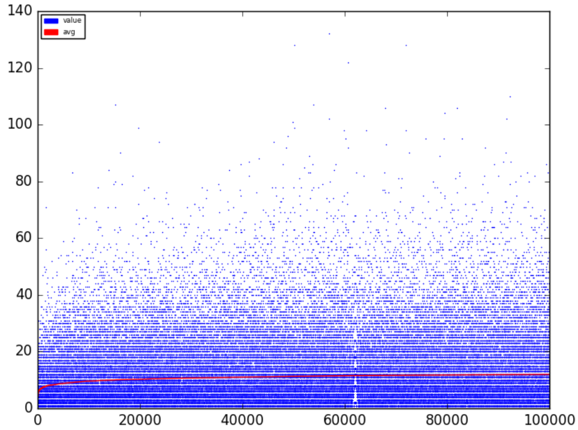


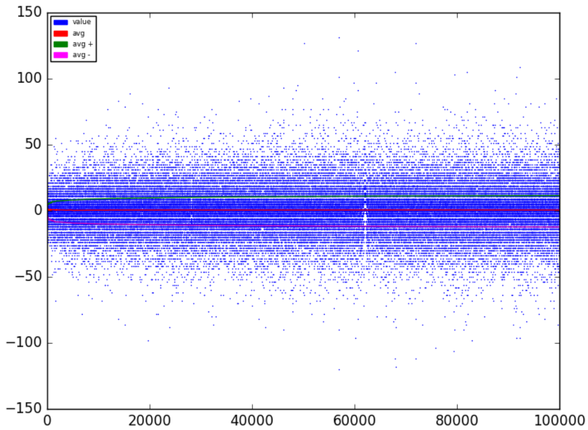
Figure 2: A333779 and match to $a \times x^b + c$ - evolution of curve fit ($0 < m < 10^7$, $m \in \mathbb{N}$)



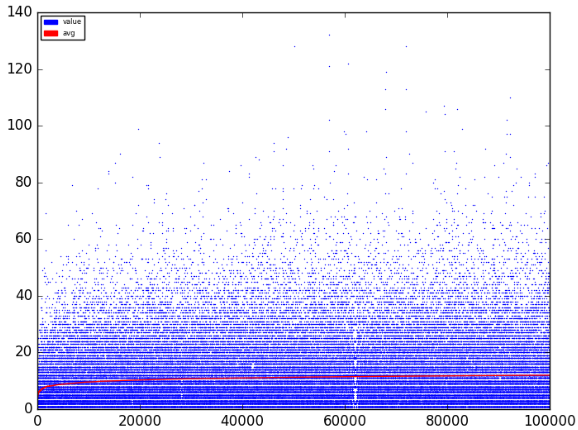
(a) Values of Δ, $Op_{\Delta} = 1$



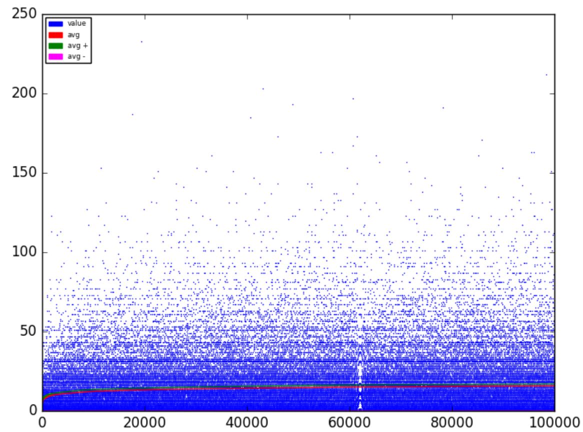
(b) Total steps required, $Op_{\Delta} = 1$



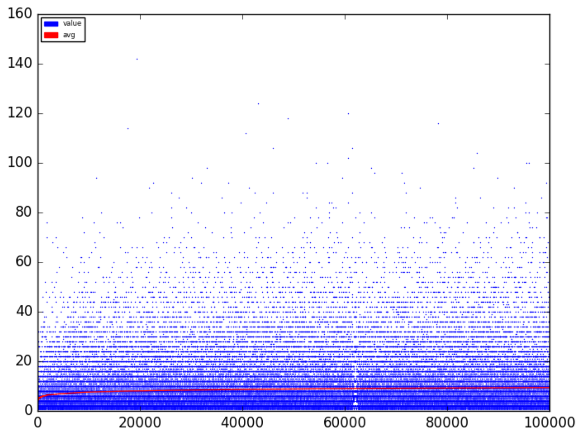
(c) Values of Δ, $Op_{\Delta} = 2$



(d) Total steps required, $Op_{\Delta} = 2$

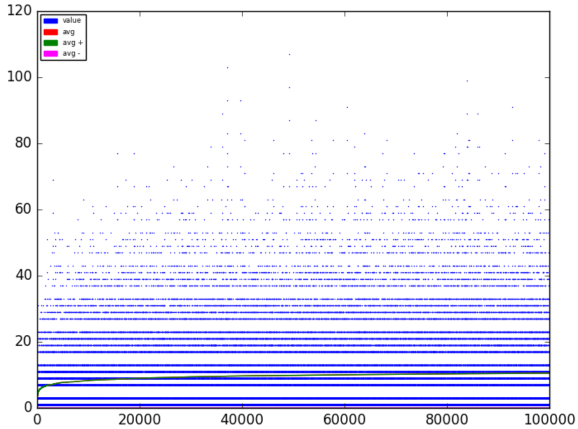


(e) Values of Δ, $Op_{\Delta} = 3$

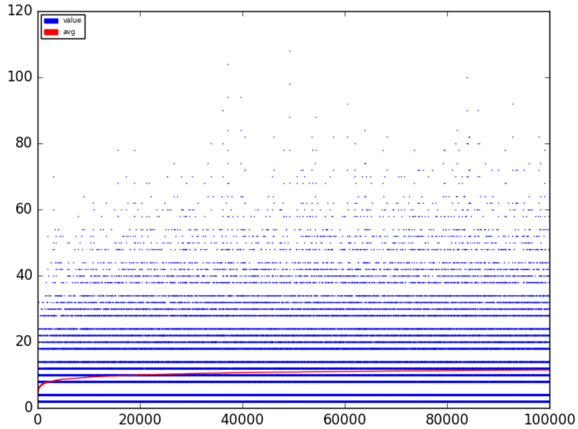


(f) Total steps required, $Op_{\Delta} = 3$

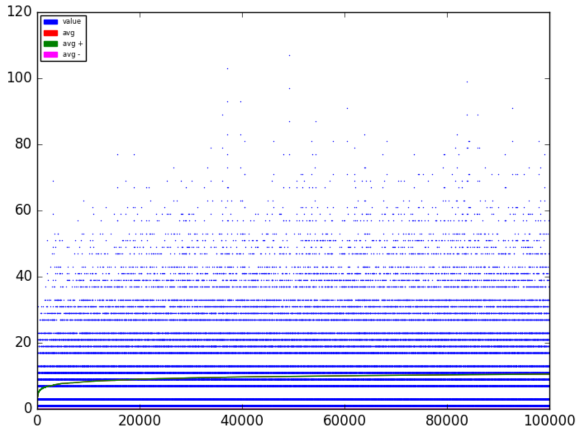
Figure 3: $0 < m \leq 10^5$, $f(m, \Delta) = 9.81764596 \times m^{1.09031708} + \Delta$



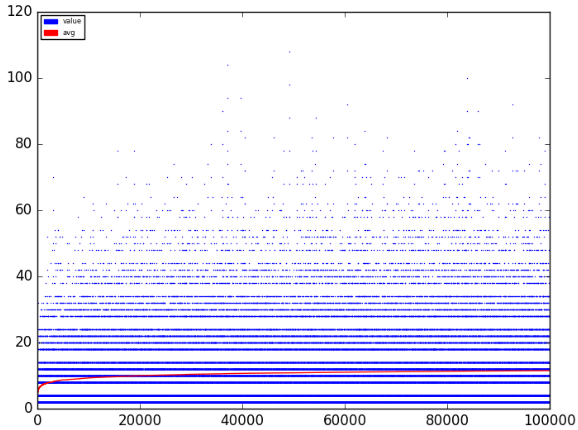
(a) Values of Δ, $Op_{\Delta} = 1$



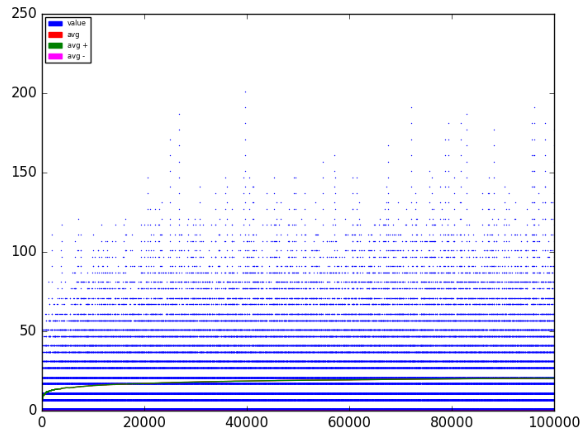
(b) Total steps required, $Op_{\Delta} = 1$



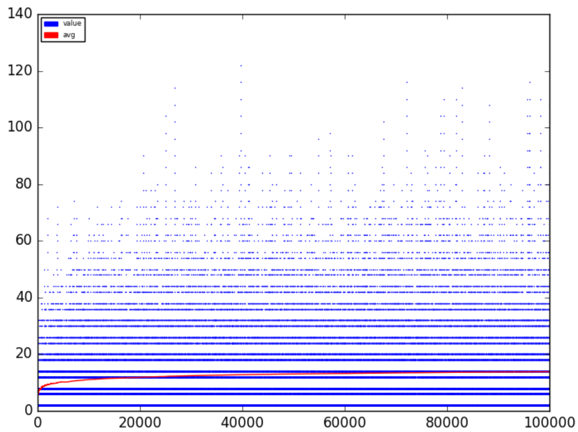
(c) Values of Δ, $Op_{\Delta} = 2$



(d) Total steps required, $Op_{\Delta} = 2$

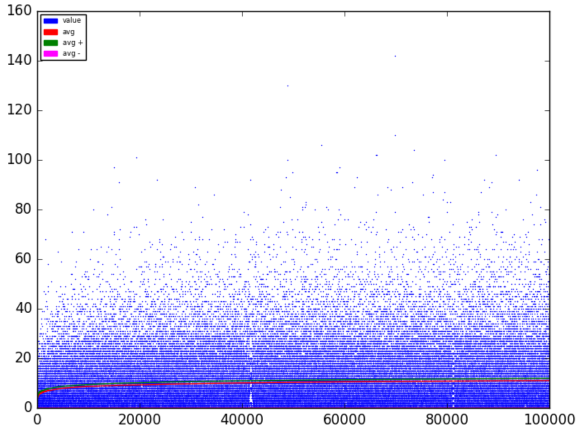


(e) Values of Δ, $Op_{\Delta} = 3$

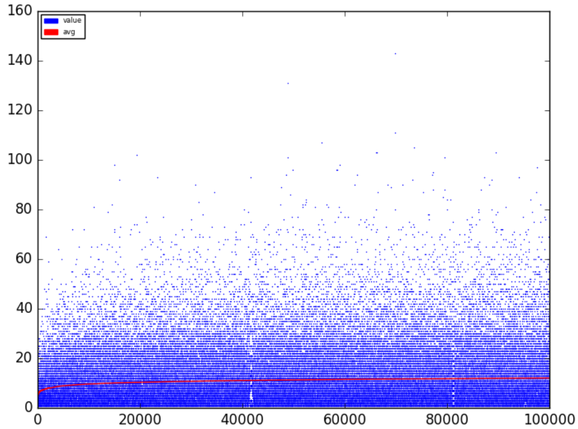


(f) Total steps required, $Op_{\Delta} = 3$

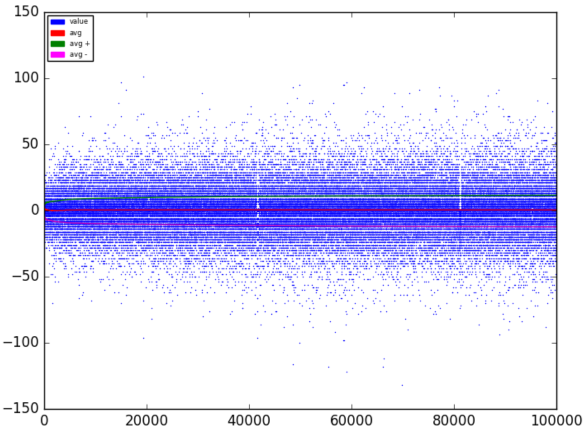
Figure 4: $0 < m \leq 10^5$, $f(m, \Delta) = 10 \times m + \Delta$



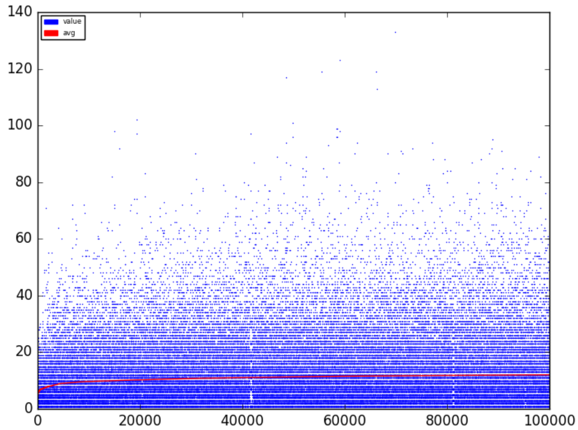
(a) Values of Δ, $Op_{\Delta} = 1$



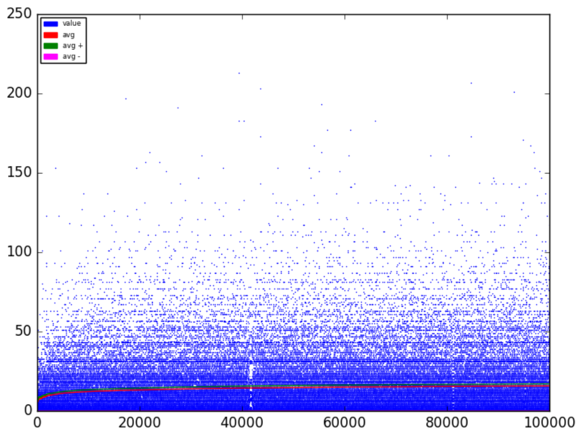
(b) Total steps required, $Op_{\Delta} = 1$



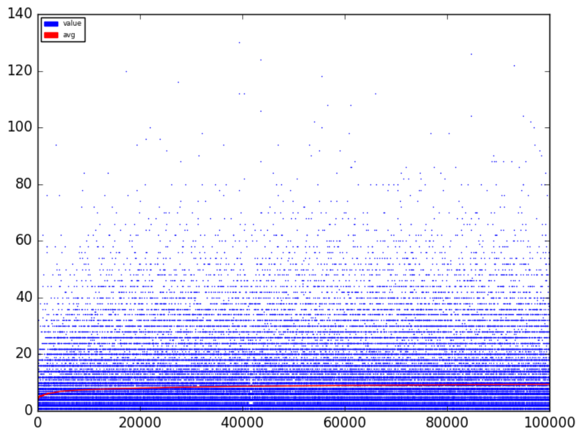
(c) Values of Δ, $Op_{\Delta} = 2$



(d) Total steps required, $Op_{\Delta} = 2$



(e) Values of Δ, $Op_{\Delta} = 3$



(f) Total steps required, $Op_{\Delta} = 3$

Figure 5: $0<m \leq 10^5$, $f(m, \Delta) = 9.1 \times m^{1.1} + 10 + \Delta$

Table 5: Experiments for $m = 1 \dots 10^5$

Op_k	Op_Δ	Greatest prime found	No. of exact matches	Average delta	Average positive delta	Average negative delta	Average number of steps
1	1	2877109	7316 (7.32%)	10.977639776397764	11.844167754604404	0	11.977639776397764
1	2	2877109	7316 (7.32%)	0.5227552275522755	11.473807193784825	-12.255467943302357	11.967459674596746
1	3	2877109	7316 (7.32%)	15.757987579875799	17.001855788008587	0	9.391663916639166
2	1	1000003	0 (0%)	10.499914999149992	10.499914999149992	0	11.499914999149992
2	2	1000003	0 (0%)	10.499914999149992	10.499914999149992	0	11.499914999149992
2	3	1000037	0 (0%)	20.454454544545445	20.454454544545445	0	13.840478404784047
3	1	2977651	7398 (7.40%)	11.027750277502776	11.90876988369456	0	12.027750277502776
3	2	2977651	7398 (7.40%)	0.44380443804438047	11.558844852585791	-12.331726627081151	12.035630356303564
3	3	2977651	7398 (7.40%)	15.748877488774887	17.007073357739117	0	9.370553705537056

4 Summary and next steps

As a result of this work two integer sequences have been submitted to OEIS database: A333779 and A333122.

References

[1] OEIS Foundation Inc. (2020), The On-Line Encyclopedia of Integer Sequences, http://oeis.org/A105093. Numbers n such that n = prime(k) + prime(k+3) = prime(k+1) + prime(k+2) for some k.

[2] OEIS Foundation Inc. (2020), The On-Line Encyclopedia of Integer Sequences, http://oeis.org/A333122. Numbers m such that m = prime(k) + prime(k+5) = prime(k+1) + prime(k+4) for some k.

[3] OEIS Foundation Inc. (2020), The On-Line Encyclopedia of Integer Sequences, http://oeis.org/A333779. Positive numbers m used to build entire prime set by m +- n without duplication.

A First 1000 terms of A333779

[2, 4, 9, 16, 27, 42, 23, 60, 51, 70, 93, 120, 85, 114, 153, 56, 165, 174, 155, 132, 213, 218, 201, 234, 253, 288, 225, 254, 135, 360, 323, 342, 315, 274, 303, 384, 395, 420, 405, 440, 357, 420, 481, 534, 465, 454, 495, 510, 515, 552, 537, 622, 699, 600, 623, 576, 663, 670, 651, 702, 679, 762, 795, 680, 705, 756, 853, 840, 729, 878, 897, 882, 911, 756, 957, 934, 957, 834, 941, 1008, 933, 352, 969, 1020, 1025, 1146, 963, 1064, 1311, 1128, 1081, 1062, 1089, 1156, 1185, 1164, 1117, 1194, 1191, 1222, 1329, 1218, 1265, 1380, 1305, 1342, 1557, 1404, 1391, 1470, 1413, 1438, 1419, 1494, 1487, 1308, 1605, 1550, 1605, 1578, 1307, 1422, 1689, 1576, 1617, 1722, 1607, 1566, 1749, 1738, 1701, 1758, 1691, 1746, 1893, 1772, 2007, 1794, 1609, 1722, 1809, 1760, 1851, 1896, 1945, 2058, 1923, 1934, 2121, 1938, 1933, 2130, 2085, 2026, 2157, 2142, 2153, 2220, 2109, 2188, 2313, 2172, 2179, 2274, 2217, 2294, 2373, 2280, 2299, 2382, 2169, 2270, 2445, 2460, 2417, 2532, 2565, 2416, 2571, 2400, 2341, 2322, 2475, 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